A Semi-Lagrangian Contouring Method for Fluid Simulation

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Overview

In this sketch we present a semi-Lagrangian surface tracking method for use with fluid simulations. Our method maintains an explicit polygonal mesh that defines the surface, and an octree data structure that provides both a spatial index for the mesh and an efficient means for evaluating the signed-distance function away from the surface. At each time step the surface is reconstructed from an implicit function defined by the composition of backward advection and the previous signed-distance function. One of the primary advantages of this formulation is that it enables tracking of surface characteristics, such as color or texture coordinates, at negligible additional cost. We include several examples demonstrating that the method can be used as part of a fluid simulation to effectively animate complex and interesting fluid behaviors.

Methods

The surface tracking problem can be phrased as: given a surface representation and a velocity field at time $t - \Delta t$, build a representation of the surface at time t. Our method represents the surface with an explicit polygonal mesh. A supplemental octree annotated with signed-distance-field samples provides a spatial index for the mesh as well as a means for approximating the signed-distance function away from the surface.

At every timestep, we build a new mesh and octree in a three-step process. First, we define a composite implicit function which relates the surface at the current timestep to the surface at the previous timestep. The function is formed by performing backwards path tracing through the flow field and then evaluating the signed distance to the previous polygonal surface. Essentially, the value of the scalar function at a point $\boldsymbol{x}$, at current time t, is obtained by tracing backward in the flow to find the previous location $\boldsymbol{x}'$ at time $t - \Delta t$, and then returning the singed distance of $\boldsymbol{x}'$ from the previous surface. When $\boldsymbol{x}'$ is near the surface, we can measure the distance to the polygonal mesh and return an *exact* signed-distance value. Second, we extract the zero-set of the composite implicit function using a marching cubes algorithm. The resulting mesh defines the surface at the current timestep. Third, a new signed-distance field is computed through an established process known as redistancing.

Our method is based in large part on prior work in the computational physics literature that describes and analyzes a method for contour tracking in two-dimensions [Strain, 2001]. We have extended the method to three-dimensions and demonstrated its use for tracking fluid surfaces.

One of the primary advantages of this method is that it enables tracking surface characteristics, such as color or texture coordinates. These properties can be easily stored directly on the polygonal mesh and efficiently mapped onto the new surface during semi-Lagrangian advection. The explicit surface representation also facilitates other common operations, such as rendering, while reconstruction from a scalar function still allows operations that rely on an implicit representation. Finally, the method produces detailed, well-defined surfaces that do not jitter or exhibit other undesirable behaviors.

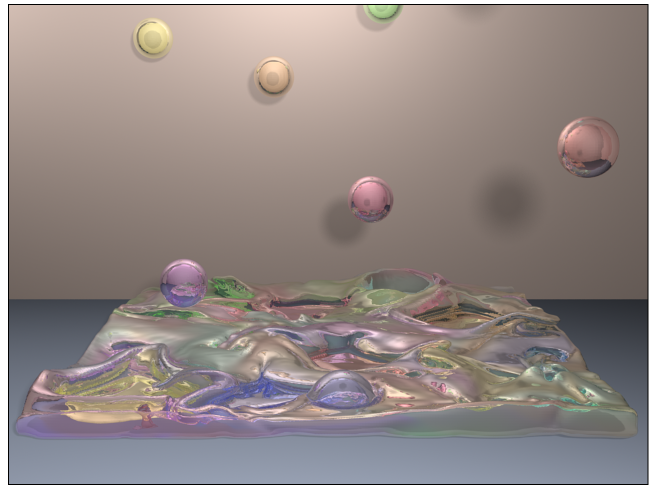

Figure 1: This image shows an invisible tank being filled as multicolored balls of viscoelastic fluid fall into it. The resulting surface contains complex geometric details which retain the different colors of the balls.

Results

We have tested this surface tracking method on a number of examples both with analytic velocity fields as well as coupled with a fluid simulation. We have found the method to be well-behaved and robust and believe it compares quite favorably to our particle level-set implementation. Since we generate a polygonal mesh for each frame we can take advantage of standard rendering techniques, allowing for very fast rendering times. Many of our examples are rendered with a matte shader so that the surface detail can be seen. A number of our examples are also rendered with refraction shaders for comparison with previous methods, with real fluids, and to demonstrate how the method can be used to generate realistic results. Our colored and textured examples illustrate how easily a variety of properties may be attached to the surface. In practice, we believe that advected properties could be used effectively with standard shading techniques to generate a wide range of interesting effects.

References

Strain, J. A. 2001. A fast semi-lagrangian contouring method for moving interfaces. *Journal of Computational Physics 169*, 1 (May), 1–22.